Bridging Units: Resource Pocket 8

Growth and Decay

Students may be familiar with the concepts of growth and decay from science lessons. This is a natural progression from the work in resource pocket 1 which looks at fractions, decimals, percentages and ratio. This pocket develops the use of decimal multipliers whilst continually refreshing the links with fractions and ratios. Students are expected to use calculators effectively for compound interest and growth and decay problems in the new GCSE, although using an iterative procedure will be examined at Higher level only. All of the resources are suitable for Foundation students.

This resource pocket progresses through three sections: developing understanding, skills builders and problem solving activities. As with all 9 resource pockets there are a number of different learning styles and approaches used to cater for a variety of learners.

1. Developing Understanding

   These are class based, teacher led questions with suggested commentary to get the most from a class or small group discussion. The boxed text can either be copied onto the whiteboard for class discussion, or printed onto cards and handed out to students to be used for paired or small group work.

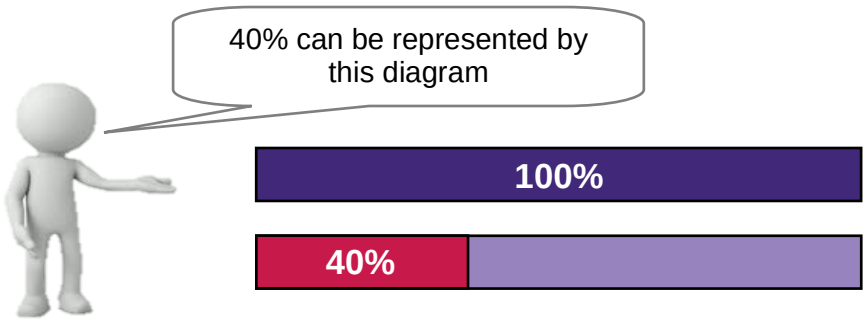
2. Skills Builders

   These are standard progressive worksheets that can be used to drill core skills in a particular area.

3. Problem Solving Activities

   Extension activities for paired work or small group work to develop problem solving skills whilst focussing on a particular area of mathematics that students can learn to apply.

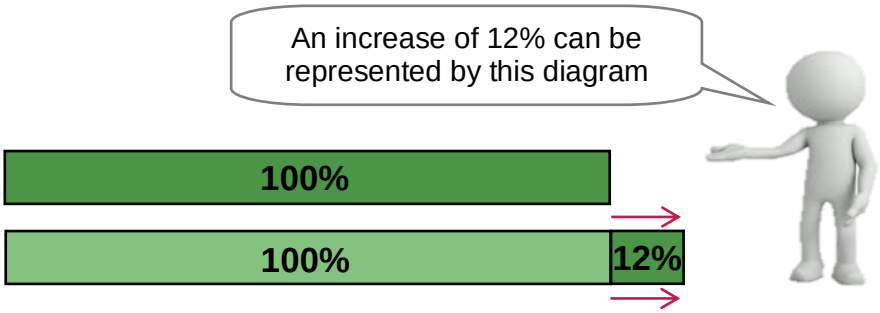
Developing Understanding 1



(a) The diagram represents 40%, what is this as a fraction?

(b) What is this as a decimal?

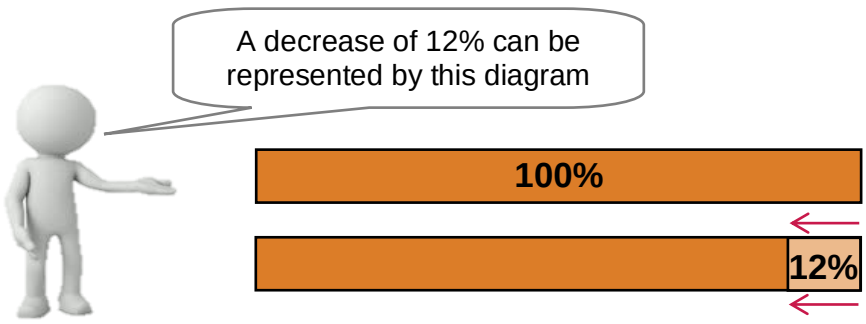
(c) Can you write a ratio for this representation?



(d) What percentage is this in total?

(e) Can you write this as a fraction?

(f) Can you write this as a decimal?



(g) What percentage is left?

(h) Can you write this as a fraction?

(i) Can you write this as a decimal?

This activity is designed to give students a consistent way of visualising increasing and decreasing by a percentage using the Singapore Bar Model.

Ensure that the students understand the representations and are confident writing these are fractions and decimals.

Some further questions could be:

- Increasing by 10% is the same as finding which percentage?
- Decreasing by 25% is the same as finding which percentage?

Answers

(a) $\frac{40}{100} = \frac{2}{5}$

(b) 0.4

(c) For example 40 : 60

(d) 112%

(e) $\frac{112}{100} + \frac{28}{25} = 1\frac{3}{25}$

(f) 1.12

(g) 88%

(h) $\frac{88}{100} = \frac{22}{25}$

(i) 0.88

Express the information shown in the pictures using:

(a) fractions

(b) decimals

(b) percentages

(c) ratios

Discuss the responses that students have written to each part, giving a special focus to ratio. This is an excellent opportunity to build on the work done in resource pockets 1 and 7 on fractions, ratio and proportion.

Tell students that the smaller packet contains 20 biscuits.

Discuss how they can use fractions, decimals, percentages and ratios to work out how many biscuits there are in the packet with 20% extra.

Make sure that all methods are covered so students can see that they all give the same answer.

Ask students to do the same calculations for an offer of $\frac{1}{4}$ extra free.

Give an extra focus to the decimal way of calculating; ensure that students understand that 1.25 is the multiplier for the increase.

Answers

For example

(a) the larger packet has $1\frac{1}{5}$ the amount of the smaller packet

(b) the larger packet has 1.2 times the amount of the smaller packet

(c) the larger packet has 20% more than the smaller packet

(d) the ratio of the amount in the smaller packet to the larger packet is 100:120 (or equivalent)

If the large packet contains 20% extra

Using fractions; $\frac{1}{5}$ of 20 biscuits is 4, so $\frac{6}{5}$ is 24 biscuits.

Using decimals; $1.2 \times 20 = 24$ biscuits

Using percentages; 20% of 20 biscuits is 4, so 120% is 24

Using ratios; one method could be 100 : 120 can simplify to 10:12. 10 parts = 20 biscuits so 12 parts = 24 biscuits.

If the large packet contains $\frac{1}{4}$ extra, then number of biscuits in the large packet will be 25.

Express the information shown in the pictures using:

(a) fractions

(b) decimals

(b) percentages

(c) ratios

Discuss the responses that students have written to each part, again giving a special focus to ratio.

Tell the students that the first house is priced at £120 000.

Discuss how they can use fractions, decimals, percentages and ratios to work out the price of the reduced house.

Again make sure that all methods are covered so students can then see that they all give the same answer.

Answers

For example

(a) the second house is $\frac{9}{10}$ the price of the first house

(b) the second house is 0.9 times the price of the first house

(c) the second house is 10% less than the first house, or is 90% of the price of the first house

(d) the ratio of the first house price to the second house price is 100:90 (or equivalent)

Using fractions; $\frac{1}{10}$ of 120,000 is 12,000 so $\frac{9}{10}$ is £108 000.

Using decimals; $0.9 \times 120{,}000 = £108{,}000$

Using percentages; 10% of 120 000 is 12,000, so 90% is £108 000

Using ratios; one method could be 100 : 90 can simplify to 10 : 9. 10 parts = £120 000 so 9 parts = £108 000.

Developing Understanding 2

Kevin is generating a number sequence using his calculator.

He starts with the number 200

To get each number in the sequence, he increases the previous number by 10%.

200, 220. 242 …

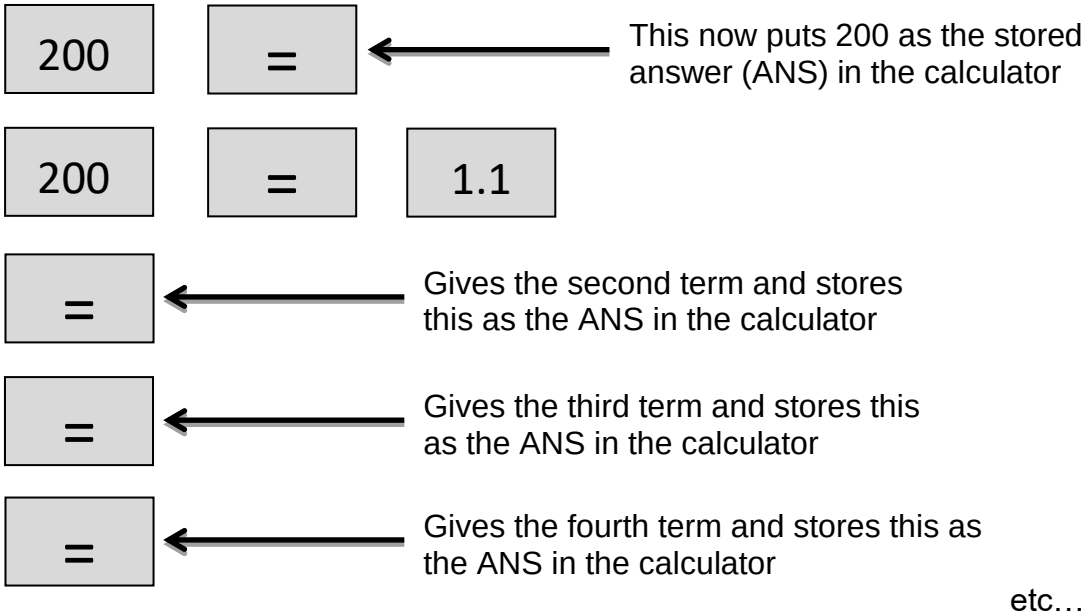
How can you generate Kevin's sequence on your calculator by pressing the MINIMUM number of buttons?

Discuss with the group how to generate the next numbers in the sequence, aiming to ensure that they can do this using a number of methods including finding 10% and adding, using a decimal multiplier or a fractional multiplier.

Using the multiplier 1.1 is the most efficient method, so allow some time for students to explore using the power button to give numbers in the sequence. Some questions that will help develop this investigation could be:

- How could we get straight to the fourth number in the sequence using a calculator?
- Which power would we use to get the third number in the sequence?
- Which power would we use to get the tenth number in the sequence?
- Can you explain how you know which power to use for any term in the sequence?

If students have not found the quickest way to generate the full sequence, instruct them to make use of the ANS button by following these instructions



Change the starting value and the percentage increased (or decreased) by and ask students to generate the sequence using this method.

Answers

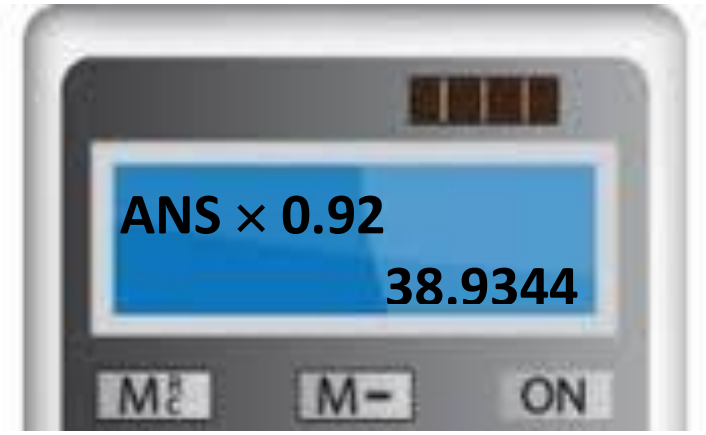
- The calculation 200×1.1^3 (= 266.2) would give the 4th number (term) in the sequence.
- To get the third number, use 200×1.1^2 so the power is 2.
- 9
- The power will always be $n - 1$ (or one less than the term number)

Sven is also creating a number sequence.

He is using the ANS button on his calculator to generate the numbers quickly.

Sven works out the fourth number in his sequence.

His calculator display says:



(a) Describe in words how Sven's sequence is generated.

(b) What was Sven's starting number?

(c) How could Sven use the power button to find the tenth number in his sequence?

Explain that although using the ANS button is the most efficient way of generating successive values, to find a particular value it is better to use the power button.

Students can use this example to set their own problems for other students to solve. They could generate their own sequence using the ANS button and by showing their partner the calculator display for a given term ask questions about the sequence similar to those above.

Answers

(a) Each number is generated by reducing the previous number by 8%
(or each number is 92% of the previous number)

(b) The starting number is 50

(c) 50×0.929 (= 23.60806816)

Developing Understanding 3

The Ginormous Green Dragon (GGD) has had a tiny baby today.

The baby measures 1 cm in length.

Each year baby GGD will double in length.

(a) On which birthday will baby GGD be over 1 metre tall for the first time?

(b) How tall will baby GGD be on his 15th birthday?

Allow students to think through these questions and then discuss as a group the different methods that were used.

Encourage students to investigate the use of the power button on the calculator to quickly get baby GGD's heights at given birthdays.

The following questions can help to fully develop the discussion:

- Do we need to write down baby GGD's height at every birthday to get his height at 15?
- In which units are the heights given on your calculator display?
- How could we change this into something more sensible?

Explain that this model can't go on forever; adults reach their maximum height and then stop growing. As a group decide on a maximum height for baby GGD.

- How long will it be before baby GGD reaches his maximum height?

Answers

(a) His 7th birthday, he will be 1.28 m tall

(b) 327.68 m

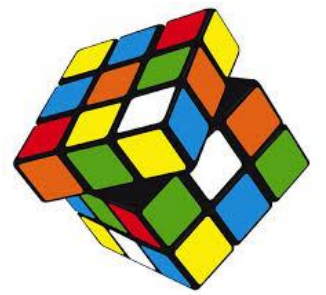
Baby GGD's height can be calculated quickly using $1 \times 2^{\text{birthday}}$

The calculator will display baby GGD's height in the same units that the initial length of 1 cm are entered. So for the calculator to display in metres, the initial length must be entered as 0.01 etc.

Kane got a Rubik's Cube for Christmas.

He practises and practises; he wants to be the Guinness World Record holder.

Initially it takes Kane 40 hours to solve the cube.



With continuous practice, Kane can reduce his time by 20% each month.

The Guinness World record for solving the Rubik's Cube is 5.55 seconds.

Will Kane ever be the record holder?

Ask the students to explore this for a while, making notes on their findings.

Encourage students to set out their work clearly with notes to explain what each calculation is for.

If needs be, the following questions will help get the investigation started:

- How can we calculate the time Kane will take after 1 month? 2 months? 3 months? etc
- In which unit is the time given on the calculator display?
- How can we change this into a more appropriate unit?
- After how many months will Kane be at World Record pace?
- What will happen the month after?
- And the month after that?
- Does this seem realistic?

Explain to students that this model has limitations as eventually Kane would be solving the cube quicker than he could even pick it up!

Answers

- The speed can be calculated using $40 \times 0.8^{\text{months passed}}$
- The units of the answer will be the same as the units used for the initial speed (if 40 is used the answer will be in hours)
- As Kane gets quicker, hours can be changed to minutes by multiplying the answer by 60 or by using an initial speed of 2400 minutes instead of 40 hours. When minutes become inappropriate, multiply the answer again by 60 to convert the time to seconds. Or use an initial start time of 144 000 seconds
- After 46 months (3 years 10 months) Kane's time will be 5.02 seconds
- After 3 years 11 months, Kane's time will be 4.01 seconds
- After 4 years, Kane's time will be 3.2 seconds
- It is getting increasingly more unrealistic.

Skills Builder 1: Understanding Multiples

Section A

(1) 14% **(2)** 26% **(3)** 8% **(4)** 2% **(5)** 23.5% **(6)** 6.5%

(7) 100% **(8)** 124% **(9)** 120% **(10)** 106% **(11)** 108.5% **(12)** 102.4%

Section B

Complete each sentence

(1) Increasing by 12% is the same as finding _____%

(2) Increasing by 35% is the same as finding _____%

(3) Increasing by 6% is the same as finding _____%

(4) Increasing by 4% is the same as finding _____%

(5) Increasing by 2.5% is the same as finding _____%

(6) Increasing by 0.5% is the same as finding _____%

(7) Decreasing by 10% is the same as finding _____%

(8) Decreasing by 45% is the same as finding _____%

(9) Decreasing by 7% is the same as finding _____%

(10) Decreasing by 1% is the same as finding _____%

(11) Decreasing by 12.5% is the same as finding _____%

(12) Decreasing by 4.2% is the same as finding _____%

Section C

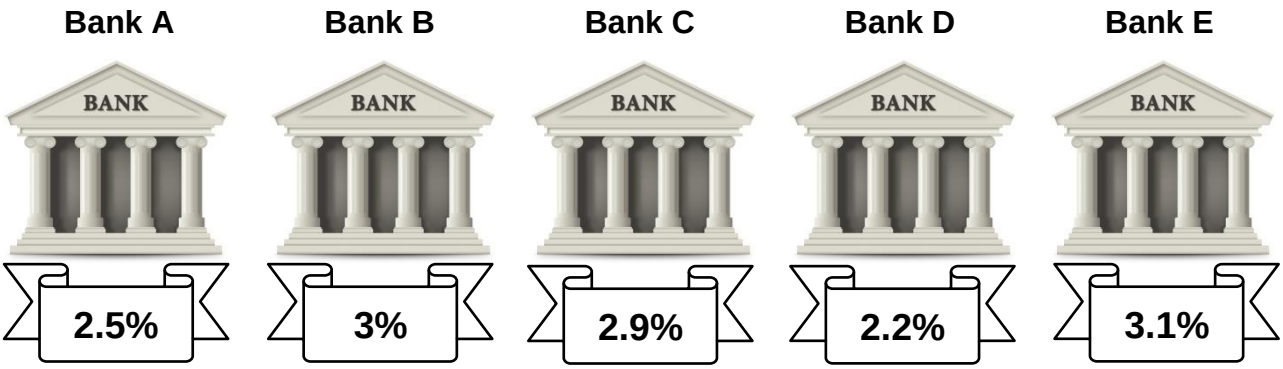
Write the multiplier that would be used to:

(1) Find 30%
(2) Find 75%
(3) Find 12%
(4) Find 5%
(5) Increase by 20%
(6) Increase by 45%
(7) Increase by 4%
(8) Increase by 2%
(9) Decrease by 10%
(10) Decrease by 25%
(11) Decrease by 8%
(12) Decrease by 1%

Skills Builder 2: Introducing Compound Inter

1 Jack has some money to invest for a year.

His five local banks are offering these annual interest rates:



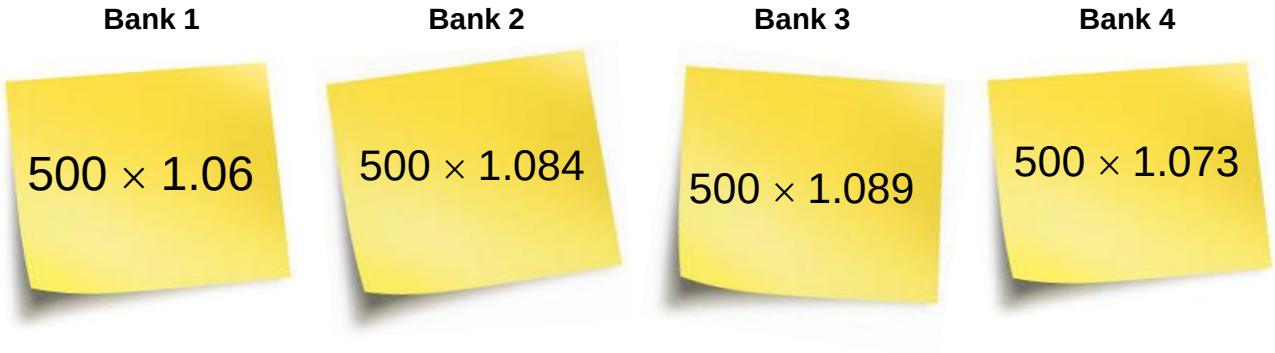
Write the multipliers that Jack would use to work out the amount of money he would have at the end of his investment:

Bank A = _____ Bank B = _____ Bank C = _____

Bank D = _____ Bank E = _____

2 Kathryn has £500 to invest for a year.

She writes the following calculations to work out the value of her investments at a number of banks:



Write down the interest rate being offered by each bank.

3 Joe invests £1000 for a year in a bank that gives 2.6% interest per annum.

(a) Using a multiplier, calculate how much Joe will receive at the end of the year.

(b) How much interest has Joe earned?

4 Kai invests £2500 for a year in a bank that gives 3.15% interest each year.

(a) Using a multiplier, calculate how much Kai will receive at the end of the year.

(b) What is the total interest Kai will earn?

5 Lola has £2000 to invest for 2 years.
The bank will pay her compound interest at 3.6% per year.

(a) Using a multiplier, work out the value of Lola's investment after the first year.

(b) Using a multiplier, work out the value of Lola's investment after the second year.

6 Seth is calculating the value of an investment he has made.
He uses this calculation:

$$180 \times 1.055 \times$$

(a) How much did Seth invest?

(b) What interest rate is Seth getting per year?

(c) How long is Seth's investment for?

(d) Is there a quicker way to put this into Seth's calculator?

7 Pav has made an investment.

He uses this calculation to work out how much his investment is worth:

$$3500 \times 1.07^2$$

(a) How much did Pav invest?

(b) What is the annual interest rate that Pav is getting?

(c) How long is Pav's investment for?

8 Match up the investment information with the correct calculation.

Give your answers as pairs of letters.

A £1500 for 2 years at a compound interest rate of 2%

B £1500 for 1 year at a compound interest rate of 2.2%

C £1500 for 2 years at a compound interest rate of 2.2%

D £1500 for 1 year at a compound interest rate of 2%

E $£1500 \times 1.022 \times 1.022$

F 1500×1.02

G 1500×1.022

H 1500×1.02^2

Skills Builder 3: Introducing Repeated Reductions

1 Louise bought a car last year for £6,500.

She now wants to sell the car but its value has reduced by 20%.

Circle the multiplier that Louise could use to work out the current value of her car.

1.2	0.2	0.8	1.02	0.98

2 Ethan is training hard for the London Marathon.
Each month his body weight reduces by 4%. His starting weight was 75 kg.

(a) Using a multiplier, calculate Ethan's body weight after the first month.

(b) Ethan says that his weight will be 69.12 kg after 2 months. Is he correct? Explain your answer.

3 A warehouse has stock worth £14 500.
After 6 months, 23% of the stock has been sold.

Using a multiplier, calculate the value of the stock left in the warehouse.

4 Claire is giving her unwanted books to a charity shop.
She had 180 books and gave 55% of them to charity.

Use a multiplier to calculate how many books Claire has left.

5 Zack owns four cars. After 1 year, three have depreciated in value.
He uses these calculations to work out the current values of his four cars:

Car A	**Car B**	**Car C**	**Car D**
			
£6000 × 0.87	£8500 × 1.04	£5800 × 0.6	£9200 × 0.83

(a) Which car has not depreciated?

(b) How has the value of this car changed?

(c) By what percentage have the other three cars depreciated?

6 The number of spectators at a football club is in decline.

Each season, ticket sales are reducing by 8%.

The total ticket sales in the first season was £5000.

(a) Use a multiplier to calculate the ticket sales in the second season.

(b) Using a multiplier, calculate the ticket sales in the third season.

7 Daley starts with £4000 savings.

Each year he will spend 20% of the amount he has at the start of the year.

(a) Use a multiplier to calculate how much money Daley will have at the start of year 2 (or at the end of year 1)

(b) Use a multiplier to calculate how much money Daley will have at the start of:

(i) year 3 (or the end of year 2)

(ii) year 4 (or the end of year 3)

(c) Daley thinks at the start of year 10 he will have more than £500.

Is he correct?

Show all of your working.

8 A rare flower grows at Pennington Flash.

This year only 600 of the flower grew.

It is predicted that each year the number of these flowers will halve.

(a) Use a multiplier to work out how many of these flowers are expected next year.

(b) In how many years is it predicted that this flower will become extinct?

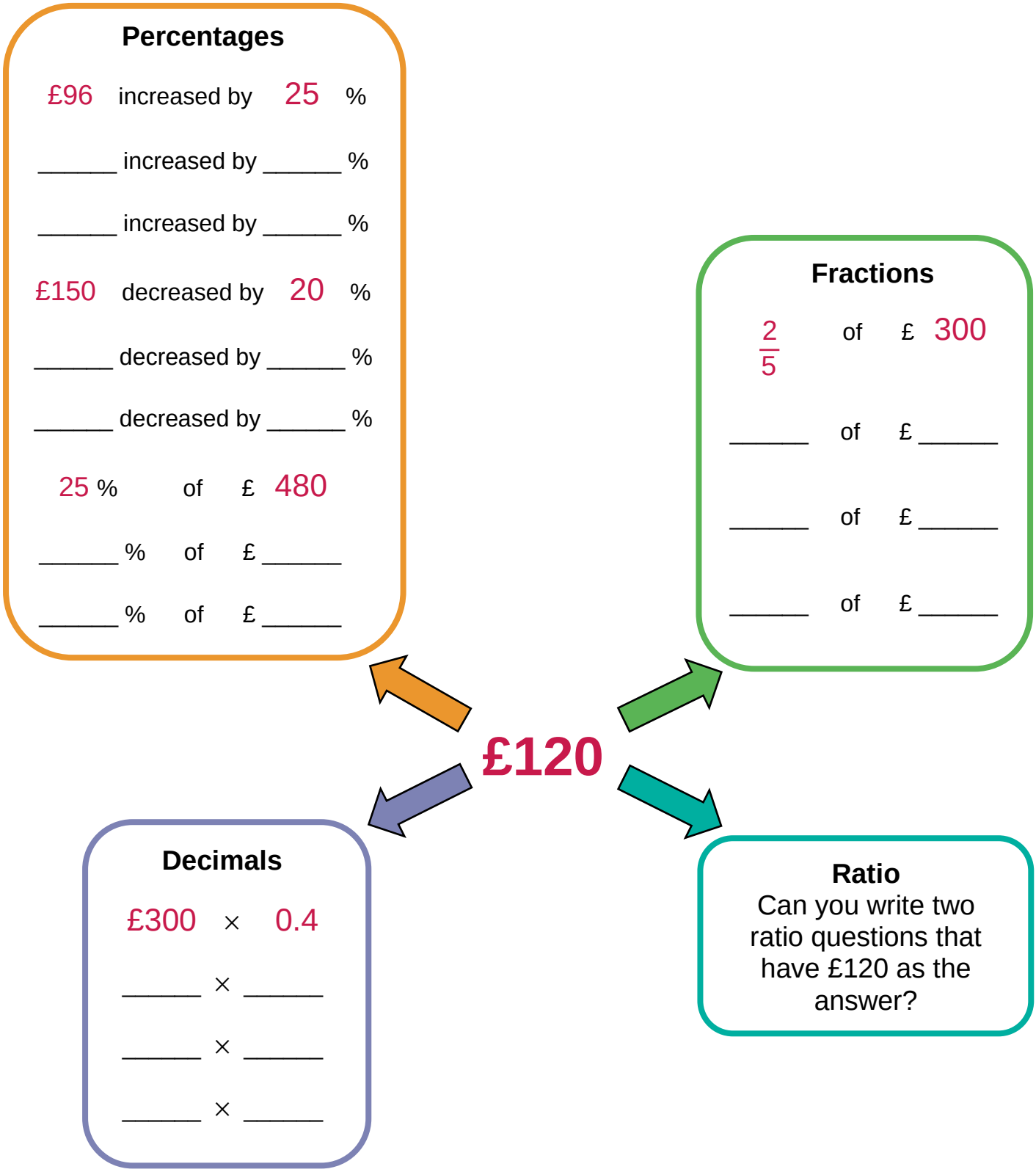
Show all of your working clearly to explain your answer.

Problem solving 1 : Percentages, Decimals, Fractions and Ratio

Complete the spider diagram.

One answer in each section has been done for you.

Try to come up with values that nobody else in the class will think of.



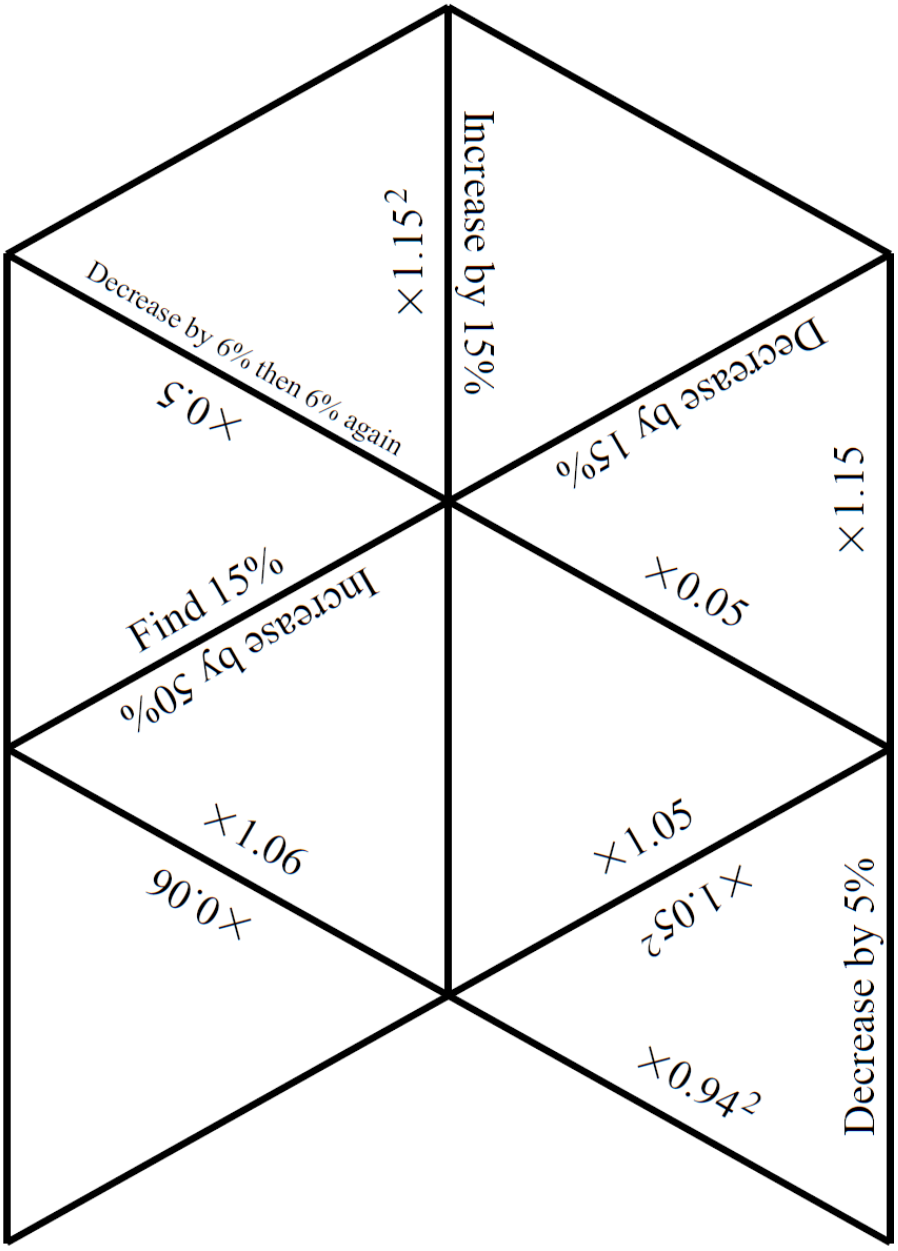
Problem solving 2 : Multipliers Tarsia

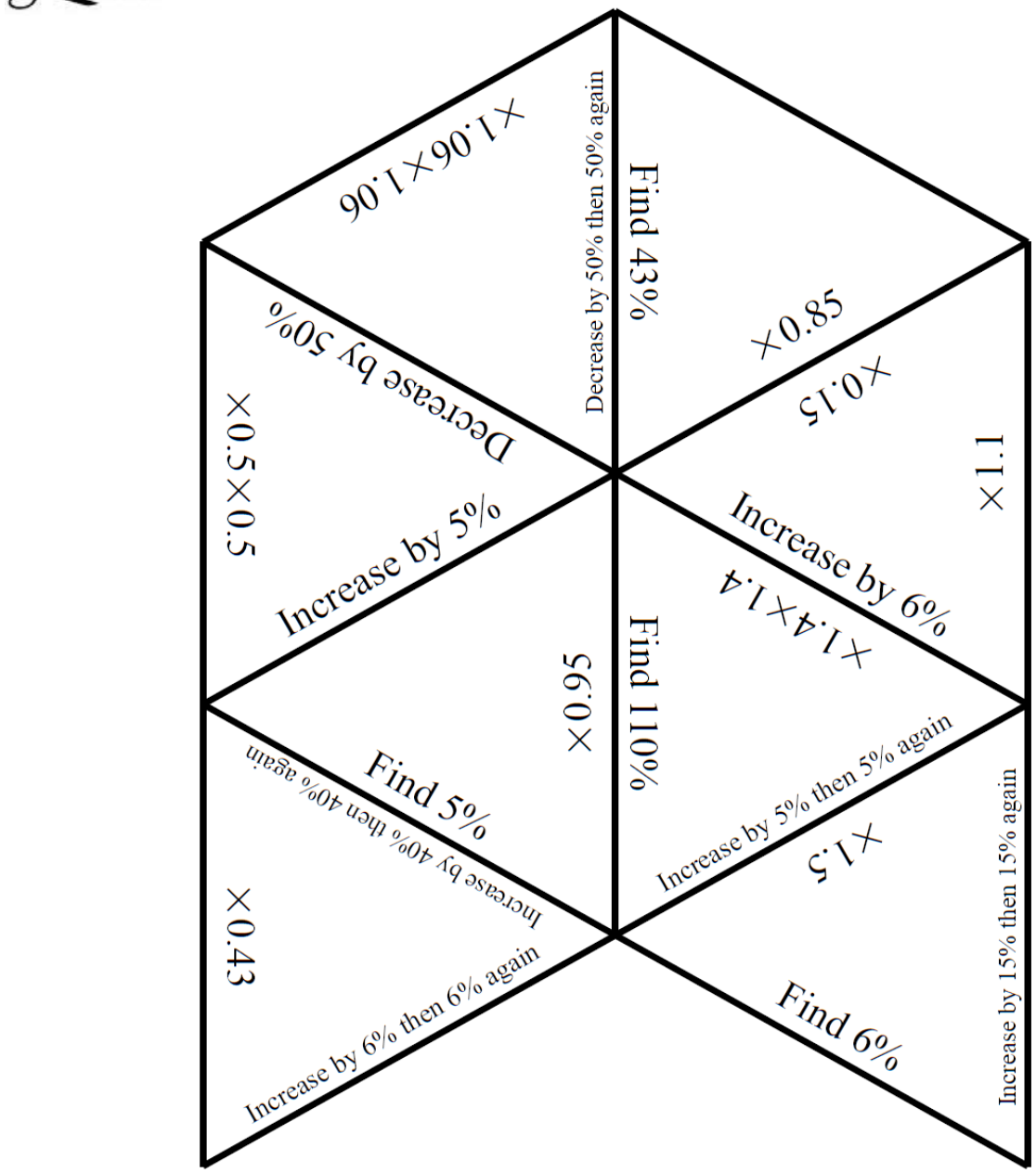
Print and cut out the cards

One full set is over TWO pages so be sure to print them both!

Give one set to each pair or group of students.

Students must match the sides to create a triangle.





Answers

Skills builder 1: Understanding Multipliers

Section A

(1) 0.14 **(2)** 0.26 **(3)** 0.08 **(4)** 0.02 **(5)** 0.235 **(6)** 0.065

(7) 1 **(8)** 1.24 **(9)** 1.2 **(10)** 1.06 **(11)** 1.085 **(12)** 1.024

Section B

(1) 112 **(2)** 135 **(3)** 106 **(4)** 104 **(5)** 102.5 **(6)** 100.5

(7) 90 **(8)** 55 **(9)** 93 **(10)** 99 **(11)** 87.5 **(12)** 95.8

Section C

(1) 0.3 **(2)** 0.75 **(3)** 0.12 **(4)** 0.05 **(5)** 1.2 **(6)** 1.45

(7) 1.04 **(8)** 1.02 **(9)** 0.9 **(10)** 0.75 **(11)** 0.92 **(12)** 0.99

Skills builder 2: Understanding Compound Interest

1 A = 1.025 B = 1.03 C = 1.029 D = 1.022 E = 1.031

2 Bank 1 = 6% Bank 2 = 8.4% Bank 3 = 8.9% Bank 4 = 7.3%

3 **(a)** £1026
(b) £26

4 **(a)** £2578.75
(b) £78.75

5 **(a)** £2072
(b) £2146.59

6 **(a)** £180
(b) 5.5%
(c) 2 years
(d) 180×1.055^2

7 **(a)** £3500
(b) 7%
(c) 2 years

8 AH, BG, CE, DF

Skills builder 3: Introducing repeated reductions

1 0.8

2 **(a)** 72 kg

(b) Yes he is correct, $75 \times 0.96 \times 0.96 = 69.12$ kg

3 £11 165

4 81

5 **(a)** Car B

(b) Increased by 4%

(c) Car A = 13% Car C = 40% Car D = 17%

6 **(a)** £4600

(b) £4232

7 **(a)** £3200

(b) (i) £2560

(ii) £2048

(c) $4000 \times 0.8^9 = £536.87$ yes he will

8 **(a)** 300

(b) In 9 years it is predicted that there will only be 1 flower so extinct in 10 years (600×0.5^9)

Problem Solving 1: Percentages, decimals, fractions and ratios

Percentages

£96 increased by 25 %

£100 increased by 20 %

£80 increased by 50 %

£150 decreased by 20 %

£240 decreased by 50 %

£200 decreased by 40 %

25 % of £ 480

80 % of £ 150

30 % of £ 400

Fractions

$\frac{2}{5}$ of £ 300

$\frac{1}{6}$ of £ 720

$\frac{5}{6}$ of £ 144

$\frac{3}{4}$ of £ 160

£120

Decimals

£300 × 0.4

£240 × 0.5

£1200 × 0.1

£600 × 0.2

Ratio

Can you write two ratio questions that have £120 as the answer?

1 Bill and Ben share £600 in the ratio 1 : 4.
How much does Bill get?

2 Jess and Jo share some money in the ratio 2 : 3.
Jo gets £180
How much does Jess get?

Problem Solving 2: Multipliers Tarsia