Franklin Sixth Form College

A Level Mathematics and Further Mathematics

Essential Bridging Work

In order to help you make the best possible start to your studies at Franklin, we have put together some bridging work that you will need to complete before you enrol. Doing your best in this work will ensure you make the most of the early weeks, which we know are really important in getting the best you can from your studies. This work must be completed to the best of your ability and handed in at your enrolment. In a sense, this is your first piece of homework and it is important to note that it will be your first piece of assessed work, it is therefore a requirement of enrolling on to your study programme.

<table>
<tr><td>Topic</td><td>Algebra
The focus of this bridging work is algebra; being able to manipulate and work with algebra fluently will give you a distinct advantage at A level. During your GCSE studies of mathematics, you would have studied algebra in various ways. At A level, algebra underpins the majority of the work you will do, so in order to be successful, you need to have experienced the skills required for the course. You may find some of these topics easy and some challenging, however, it is vital you attempt all of it and use the videos to guide you through the more difficult sections.</td></tr>
<tr><td>Task</td><td>A Level Mathematics

Complete the booklet (ignoring any ‘further maths only’ sections) using the following criteria:

• Use lined paper to complete each exercise
• Neatly and clearly, show all working out (use the examples as a guide).
• Use the videos at the end of the examples to assist you.

Further Mathematics

Complete the booklet using the following criteria:

• Use lined paper to complete each exercise
• Neatly and clearly, show all working out (use the examples as a guide).
• Use the videos at the end of the examples to assist you.
• Complete all exercises including the ‘further maths only’ sections.</td></tr>
<tr><td>Resources</td><td>Online Research

Video links are given in the booklet; however, feel free to do your own independent study. There are many excellent Youtube channels and websites, such as:

• Exam Solutions - https://www.youtube.com/user/ExamSolutions
• Hegarty Maths - https://www.youtube.com/user/HEGARTYMATHS
• Maths Genie - https://www.mathsgenie.co.uk/gcse.html
• MyMaths (if you have access) - https://www.mymaths.co.uk/</td></tr>
</table>

	The following book covers all of the topics in the booklet and further topics that you will encounter during the A level course. We advise that you purchase this at the very reasonable price of £5.95. https://www.amazon.co.uk/Head-Start-Level-Maths-2017-2018/dp/1782947922
Presentation	You are required to bring this work to your enrolment with all 5 exercises within the booklet to be fully completed. As mentioned above, your working-out should be neat and clear, using the examples as a guide to help you. We look forward to reviewing your preparation work for mathematics.

Bridging Work

Name: ……………………………………………………………

Welcome to A Level mathematics!

This is the bridging booklet which will enable you to consolidate your mathematical skills ready for A level mathematics and A level further mathematics. Many students find A level mathematics a challenge compared with GCSE mathematics regardless of their grade.

The work in the following pages is not designed to teach new skills but rather to hone your skills. Hopefully, these skills are not new to you. We recognise that students, due to various factors, will possibly not be 100% confident with these skills. A huge difference between A level and GCSE is the manner of the answers, A level expects a full explanation of working, showing all steps of calculation. Please don't look for shortcuts whilst working through this booklet. Show all steps of calculation neatly, we often say "Would you be proud of this work going on the wall?".

The main focus of this bridging work is algebra; being able to manipulate and work with algebra fluently will give you a distinct advantage at A level. Any 'gaps' you find in your knowledge we would expect you to independently practise those skills before the course – there are many excellent websites (YouTube videos are extremely helpful) where you can find the required practise. Along the same lines, letting your college tutor know early in the course of any skills you have struggled with will benefit you and your tutor. Attempt all the work and really work hard during the summer to master the techniques.

On a final note, just because you *can* multiply out brackets, it doesn't mean that you *should*. Indeed, at this level it is often better to keep an expression in its factorised (bracketed) form.

Good luck!

Topic	**Done Exercise (✓)**	**☺ / 😐 / ☹**
1.1 - Simple algebraic expressions		
1.2 - Algebraic fractions		
1.3 - Quadratic expressions		
1.4 - Cancelling		
1.5 - Fractional and negative powers, and surds		

Algebra

Many people dislike algebra; for many it is the point at which they start switching off mathematics. But do persevere – most of it is natural enough when you think about it the right way.

1.1 Simple algebraic expressions

Some very basic things here, but they should prove helpful.

Are you fully aware that $\frac{x}{4}$ and $\frac{1}{4}x$ are the same thing?

Example 1 Find the value of *a* for which $\frac{8}{11}(5x-4)=\frac{8(5x-4)}{a}$ is always true.

Solution Dividing 8 by 11 and multiplying by $(5x – 4)$ is the same as multiplying 8 by $(5x – 4)$ and dividing by 11. So $a = 11$.

You do not need to multiply anything out to see this!

Remember that in algebraic fractions such as $\frac{3}{x-2}$, the line has the same effect as a bracket round the denominator. You may well find it helpful actually to *write in* the bracket: $\frac{3}{(x-2)}$.

Example 2 Solve the equation $\frac{3}{x-2}=12$.

Solution

Multiply both sides by $(x – 2)$:	$3 = 12(x – 2)$
Multiply out the bracket:	$3 = 12x – 24$
Add 24 to both sides:	$27 = 12x$
Divide by 12:	$x=\frac{27}{12}=2\frac{1}{4}$.

A common mistake is to start by dividing by 3. That would give $\frac{1}{x-2}=4$ [*not* $x – 2 = 4$] and you will still have to multiply by $(x – 2)$.

Don't ever be afraid to get the x-term on the *right*, as in the last line but one of the working. After all, $27 = 12x$ means just the same as $12x = 27$

Example 4 Solve the equation $\frac{3}{5}(2x+3)=\frac{7}{15}(4x-9)$

Solution Do ***not*** multiply out the brackets to get fractions – that leads to horrible numbers! Instead:

Multiply both sides by 15: $15\times\frac{3}{5}(2x+3)=15\times\frac{7}{15}(4x-9)$

Choose 15 as it gets rid of all the fractions.

Cancel down the fractions: $3\times\frac{3}{1}(2x+3)=\frac{7}{1}(4x-9)$

$$9(2x+3)=7(4x-9)$$

Now multiply out: $18x + 27 = 28x - 63$

$$90 = 10x$$

Hence the answer is $x = 9$

This makes the working very much easier. ***Please don't*** respond by saying "well, my method gets the same answer"! You want to develop your flexibility and your ability to find the easiest method if you are to do well at A Level, as well as to be able to use similar techniques in algebra instead of numbers. It's not just this example we are worried about – it's more complicated examples of a similar type.

Youtube Videos to help!

Solving Equations with variables on both sides- https://www.youtube.com/watch?v=Y9U0L__kV64

Solving Equations with variables on both sides- https://www.youtube.com/watch?v=DiUzSTm330U

Changing the subject of a formula - https://www.youtube.com/watch?v=IDBAzbFTQxY

Exercise 1.1

1 Find the values of the letters p, q and r that make the following pairs of expressions always equal.

(a) $\frac{1}{7}x = \dfrac{x}{p}$ (b) $\frac{1}{5}(2x+3) = \dfrac{(2x+3)}{q}$ (c) $\frac{3}{10}(2-7x) = \dfrac{3(2-7x)}{r}$

2 Solve the following equations.

(a) $\dfrac{60}{x+4} = 12$ (b) $\dfrac{35}{2x-3} = 5$ (c) $\dfrac{20}{6-x} = \dfrac{1}{2}$

3 Make $\cos C$ the subject of the formula $c^2 = a^2 + b^2 - 2ab\cos C$.

4 (a) Multiply $\dfrac{x+5}{4}$ by 8. (b) Multiply $(x + 2) \div 3$ by 12.

(c) Multiply $\frac{1}{2}(x+7)$ by 6. (d) Multiply $\frac{1}{4}(x-3)$ by 8.

5 Solve the following equations.

(a) $\frac{3}{4}(2x+3) = \frac{5}{8}(x-2)$ (b) $\frac{1}{6}(5x+11) = \frac{2}{3}(2x-4)$

(c) $\frac{5}{9}(3x+1) = \frac{7}{12}(2x+1)$

6 Make x the subject of the following equations.

(a) $\dfrac{a}{b}(cx+d) = x+2$ (b) $\dfrac{a}{b}(cx+d) = \dfrac{2a}{b^2}(x+2d)$

7 Simplify the following as far as possible.

(a) $\dfrac{a+a+a+a+a}{5}$ (b) $\dfrac{b+b+b+b}{b}$

(c) $\dfrac{c\times c\times c\times c}{c}$ (d) $\dfrac{d\times d\times d\times d}{4}$

1.2 Algebraic Fractions

Many people have only a hazy idea of fractions. That needs improving if you want to go a long way with maths – you will need to be confident in handling fractions consisting of letters as well as numbers.

Remember, first, how to multiply a fraction by an integer. You multiply only the top *[what happens if you multiply both the top and the bottom of a fraction by the same thing?]*

Example 1 Multiply $\frac{3x}{7y}$ by 2.

Solution $3 \times 2 = 6x$, so the answer is $\frac{6x}{7y}$. (*Not* $\frac{6x}{14y}$!)

Example 2 Divide $\frac{3y^2}{4x}$ by y.

Solution $\frac{3y^2}{4x} \div y = \frac{3y^2}{4x} \times \frac{1}{y} = \frac{3y^2}{4xy} = \frac{3y}{4x}$, so the answer is $\frac{3y}{4x}$. [Don't forget to simplify.]

Double fractions, or mixtures of fractions and decimals, are always wrong.

For instance, if you want to divide $\frac{xy}{z}$ by 2, you should not say $\frac{0.5xy}{z}$ but $\frac{xy}{2z}$. This sort of thing is extremely important when it comes to rearranging formulae.

Example 3 Make r the subject of the equation $V = \frac{1}{2}\pi r^2 h$.

Solution Multiply by 2: $2V = \pi r^2 h$

Don't "divide by $\frac{1}{2}$".

Divide by π and h: $\frac{2V}{\pi h} = r^2$

Square root both sides: $r = \sqrt{\frac{2V}{\pi h}}$.

You should *not* write the answer as $\sqrt{\frac{V}{\frac{1}{2}\pi h}}$ or $\sqrt{\frac{2V}{\pi} \div h}$, as these are fractions of fractions.

Make sure, too, that you write the answer properly. If you write $\sqrt{}2V/\pi h$ it's not at all clear that the whole expression has to be square-rooted and you will lose marks.

You will often want to combine two algebraic expressions, one of which is an algebraic fraction, into a single expression. You will no doubt remember how to add or subtract fractions, using a common denominator.

Example 4 Simplify $\frac{3}{x-1}-\frac{1}{x+1}$.

Solution Use a common denominator. [You must treat $(x-1)$ and $(x+1)$ as separate expressions with no common factor.]

$$\frac{3}{x-1}-\frac{1}{x+1}=\frac{3(x+1)-(x-1)}{(x-1)(x+1)}$$

$$=\frac{3x+3-x+1}{(x-1)(x+1)}=\frac{2x+4}{(x-1)(x+1)}.$$

Do use brackets, particularly on top – otherwise you are likely to forget the minus at the end of the numerator (in this example subtracting -1 gives +1).

Don't multiply out the brackets on the bottom. You will need to see if there is a factor which cancels out (although there isn't one in this case).

Example 5 Write $\frac{3}{x+1}+2$ as a single fraction.

Solution

$$\frac{3}{x+1}+2=\frac{3}{x+1}+\frac{2}{1}$$

$$=\frac{3+2(x+1)}{x+1}$$

$$=\frac{2x+5}{x+1}$$

This method often produces big simplifications when roots are involved

Youtube Videos to help!

Simplifying Algebraic Fractions – https://www.youtube.com/watch?v=3j2ghl_tV3Q

Adding/Subtracting Alegbraic Fractions - https://www.youtube.com/watch?v=jgGBdTL-OUw

Exercise 1.2

1 Work out the following. Answers *may* be left as improper fractions.

(a) $\frac{4}{7}\times 5$ (b) $\frac{5}{12}\times 3$ (c) $\frac{7}{9}\times 2$ (d) $\frac{4}{15}\times 3$

(e) $\frac{8}{11}\div 4$ (f) $\frac{8}{11}\div 3$ (g) $\frac{6}{7}\div 3$ (h) $\frac{6}{7}\div 5$

(i) $\frac{3x}{y}\times x$ (j) $\frac{3x}{y^2}\times y$ (k) $\frac{5x^3}{4y}\div x$ (l) $\frac{5x^2}{6y}\div y$

(m) $\frac{5x^3}{2y}\times 3x$ (n) $\frac{3y^4}{4x^2z}\times 2x$ (o) $\frac{6x^2y^3}{5z}\div 2xy$ (p) $\frac{5a^2}{6x^3z^2}\div 2y$

2 Make x the subject of the following formulae.

(a) $\frac{1}{2}A = \pi x^2$ (b) $V=\frac{4}{3}\pi x^3$ (c) $\frac{1}{2}(u+v) = tx$ (d) $W=\frac{2}{3}\pi x^2 h$

3 Write as single fractions.

(a) $\frac{2}{x-1}+\frac{1}{x+3}$ (b) $\frac{2}{x-3}-\frac{1}{x+2}$ (c) $\frac{1}{2x-1}-\frac{1}{3x+2}$ (d) $\frac{3}{x+2}+1$

(e) $2-\frac{1}{x-1}$ (f) $\frac{2x}{x+1}-3$ (g) $\frac{3}{4(2x-1)}-\frac{1}{4x^2-1}$

Further Maths Only

4* Write as single fractions.

(a) $\frac{x+1}{\sqrt{x}}+\sqrt{x}$ (b) $\frac{2x}{\sqrt{x+3}}+\sqrt{x+3}$ (c) $\frac{x}{\sqrt[3]{x-2}}+\sqrt[3]{(x-2)^2}$

1.3 Quadratic Expressions

You will no doubt have done much on these for GCSE. But they are so prominent at A Level that it is essential to make sure that you are never going to fall into any traps.

First, a reminder that (a) $(x + 3)^2$ is ***not*** equal to $x^2 + 9$

(b) $\sqrt{x^2 + y^2}$ is ***not*** equal to $x + y$.

If you always remember that "square" means "multiply by itself" you will remember that $(x+3)^2 = (x+3)(x+3) = x^2 + 3x + 3x + 9 = x^2 + 6x + 9$.

A related process is to write a quadratic expression such as $x^2 + 6x + 11$ in the form $(x+a)^2 + b$. This is called ***completing the square***. Completing the square for quadratic expressions in which the coefficient of x^2 is 1 is very easy. The number a inside the brackets is always half of the coefficient of x.

Example 1 Write $x^2 + 6x + 4$ in the form $(x + a)^2 + b$.

Solution $x^2 + 6x + 4 = (x + 3)^2 – 9 + 4$

$= (x + 3)^2 – 5$.

This version immediately gives us several useful pieces of information. For instance, we now know a lot about the graph of $y = x^2 + 6x + 4$:

- It is a translation of the graph of $y = x^2$ by 3 units to the left and 5 units down
- Its line of symmetry is $x = –3$
- Its lowest point or vertex is at $(–3, –5)$

And we can solve the equation $x^2 + 6x + 4 = 0$ *exactly* without having to use the quadratic equation formula, to locate the roots of the function:

$x^2 + 6x + 4 = 0$

$\Rightarrow \quad (x + 3)^2 – 5 = 0$

$\Rightarrow \quad (x + 3)^2 = 5$

$\Rightarrow \quad x = –3 \pm \sqrt{5}$ [don't forget that there are two possibilities!]

<u>Youtube Videos to help!</u>

Completing the Square - https://www.youtube.com/watch?v=V65M4xNkCDs

Factorising Quadratics - https://www.youtube.com/watch?v=CLkoODzshoU

The difference of two squares - https://www.youtube.com/watch?v=Xx4kPp3SnzY

Exercise 1.3

1 Write without brackets.

(a) $(x + 5)^2$ (b) $(3x - 2)^2$ (c) $(3x + 4)(3x - 4)$

2 Simplify the following equations into the form $ax + by + c = 0$.

(a) $(x + 3)^2 + (y + 4)^2 = (x - 2)^2 + (y - 1)^2$

(b) $(2x + 1)^2 + (y - 3)^2 = (2x + 3)^2 + (y + 1)^2$

3 Simplify the following where possible.

(a) $\sqrt{x^2 + 4}$ (b) $\sqrt{x^2 - 4x + 4}$ (c) $\sqrt{x^2 - 1}$

(d) $\sqrt{x^2 + 9x}$ (e) $\sqrt{x^2 - y^2}$ (f) $\sqrt{x^2 + 2xy + y^2}$

4 Write the following in the form $(x + a)^2 + b$.

(a) $x^2 + 8x + 19$ (b) $x^2 - 10x + 23$ (c) $x^2 - 5x - 6$

5 Factorise as fully as possible.

(a) $x^2 - 25$ (b) $4x^2 - 36$ (c) $4x^2 - 9y^4$

(d) $3x^2 - 7x + 2$ (e) $3x^2 - 5x + 2$ (f) $6x^2 - 5x - 6$

Further Maths Only

6* Multiply out and simplify.

(a) $\left(x + \frac{1}{x}\right)^2$ (b) $\left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$ (c) $\left(x + \frac{2}{x}\right)\left(x - \frac{3}{x}\right)$

1.4 Cancelling

The word "cancel" is a very dangerous one. It means two different things, one safe enough and the other very likely to lead you astray.

You can cancel *like terms* when they are added or subtracted.

Example 1 Simplify $(x^2 - 3xy) + (3xy - y^2)$.

Solution $(x^2 - 3xy) + (3xy - y^2) = x^2 - \cancel{3xy} + \cancel{3xy} - y^2 = x^2 - y^2$.

The "$3xy$" terms have "cancelled out". This is safe enough.

It is also usual to talk about "cancelling down a fraction". Thus $\frac{10}{15} = \frac{2}{3}$. However, this tends to be very dangerous with anything other than the most straightforward numerical fractions. Consider, for instance, a fraction such as $\dfrac{x^2+2xy}{xy+2y^2}$. If you try to "cancel" this, you're almost certain not to get the right answer, which is in fact $\dfrac{x}{y}$ (as we will see in Example 2, below).

Example 2 Simplify $\dfrac{x^2+2xy}{xy+2y^2}$.

Solution Factorise the top as $x(x + 2y)$ and the bottom as $y(x + 2y)$:

$$\frac{x^2+2xy}{xy+2y^2} = \frac{x(x+2y)}{y(x+2y)}$$

Now it is clear that both the top and the bottom have a factor of $(x + 2y)$.

So this can be divided out to give the answer of $\dfrac{x}{y}$.

Don't "cancel down". Factorise if you can; divide all the top and all the bottom.

Try instead to use the word "divide". What happens when you "cancel down" $\frac{10}{15}$ is that you *divide top and bottom* by 5. If you can divide both the top and bottom of a fraction by the same thing, this is a correct thing to do and you will get a simplified answer.

Taking out factors

I am sure you know that $7x^2 + 12x^3$ can be factorised as $x^2(7 + 12x)$.

You should be prepared to factorise an expression such as $7(x + 2)^2 + 12(x + 2)^3$ in the same way.

Example 3 Factorise $7(x + 2)^2 + 12(x + 2)^3$

Solution $7(x + 2)^2 + 12(x + 2)^3 = (x + 2)^2(7 + 12(x + 2))$

$$= (x + 2)^2(12x + 31).$$

The only differences between this and $7x^2 + 12x^3$ are that the common factor is $(x + 2)^2$ and not x^2; and that the other factor, here $(7 + 12(x + 2))$, can be simplified.

If you multiply out the brackets you will get a cubic and you will have great difficulty in factorising that. ***Don't multiply out brackets if you can help it!***

Youtube Videos to help!

Simplifying algebraic fractions - https://www.youtube.com/watch?v=tlKN8NNNxdI

Simplifying complex fractions 1 - https://www.youtube.com/watch?v=qcGQIMRCvsM

Simplifying complex fractions 2 - https://www.youtube.com/watch?v=4p2FN3ib7is

The last 2 videos here are quite complex – these are particularly advisable for students who wish to undertake further mathematics.

Exercise 1.4

1 Simplify the following as far as possible.

(a) $5x + 3y + 7x - 3y$

(b) $3x^2 + 4xy + y^2 + x^2 - 4xy - y^2$.

(c) $\dfrac{4+6x}{2}$

(d) $\dfrac{4\times 6x}{2}$

(e) $\dfrac{3x+xy}{x}$

(f) $\dfrac{4x+9y}{2x+3y}$

(g) $\dfrac{4x+6y}{6x+9y}$

(h) $\dfrac{5xy+6y^2}{10x+12y}$

(i) $\dfrac{3x^2+4y^2}{6x^2-8y^2}$

(j) $\dfrac{x-3}{3-x}$

(k) $\dfrac{x^2-2xy-y^2}{y^2+2xy-x^2}$

2 Make x the subject of the following formulae.

(a) $\dfrac{ax}{b} = \dfrac{py}{qz}$

(b) $\dfrac{3\pi ax}{b} = \dfrac{4y^2}{qz}$

3 Simplify the following.

(a) $\dfrac{2\pi x}{ab} \div \frac{1}{3}\pi r^3$

(b) $\dfrac{2\pi h^2}{rb} \div \frac{4}{3}\pi hr^2$

4 Simplify into a single factorised expression.

(a) $(x-3)^2 + 5(x-3)^3$

(b) $4x(2x+1)^3 + 5(2x+1)^4$

(c)* $\frac{1}{2}k(k+1)+(k+1)$

(d)* $\frac{1}{6}k(k+1)(2k+1)+(k+1)^2$

5 Simplify as far as possible.

(a) $\dfrac{x^2+6x+8}{x^2-x-6}$

(b) $\dfrac{3x^2-2x-8}{x^2-4}$

(c) $\dfrac{(x+3)^2-2(x+3)}{x^2+2x-3}$

(d) $\dfrac{x(2x-1)^2-x^2(2x-1)}{(x-1)^2}$

Further Maths Only

(e)* $\dfrac{\dfrac{x^2}{\sqrt{x^2+1}}-\sqrt{x^2+1}}{x^2}$

(f)* $\dfrac{-\dfrac{x}{2\sqrt{1-x}}+\sqrt{1-x}}{x^2}$

1.6 Fractional and negative powers, and surds

This may seem a rather difficult and even pointless topic when you meet it at GCSE, but you will soon see that it is extremely useful at A Level, and you need to be confident with it.

> ***Negative*** powers give ***reciprocals*** (1 over the power).
>
> ***Fractional*** powers give ***roots*** (such as $\sqrt[3]{x}$).
>
> $\boldsymbol{x^0 = 1}$ for any x (apart from 0^0 which is undefined).

Examples 1 (a) $\frac{1}{x^3} = x^{-3}$ (b) $\sqrt[3]{x} = x^{\frac{1}{3}}$ (c) $\pi^0 = 1$

(d) $\sqrt[4]{x^7} = x^{7/4}$. The easiest way of seeing this is to write it as $(x^7)^{\frac{1}{4}}$

You will make most use of the rules of ***surds*** when checking your answers! An answer that you give as $\frac{6}{\sqrt{3}}$ will probably be given in the book as $2\sqrt{3}$, and $\frac{2}{3-\sqrt{7}}$ as $3+\sqrt{7}$. Before worrying why you have got these wrong, you should check whether they are equivalent!

Examples 2

Indeed, they are, as

$$\frac{6}{\sqrt{3}} = \frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$$

and

$$\frac{2}{3-\sqrt{7}} = \frac{2}{3-\sqrt{7}} \times \frac{3+\sqrt{7}}{3+\sqrt{7}} = \frac{2(3+\sqrt{7})}{3^2-(\sqrt{7})^2} = \frac{2(3+\sqrt{7})}{9-7} = 3+\sqrt{7}\,.$$

The first of these processes is usually signalled by the instruction “write in surd form” and the second by “rationalise the denominator”.

Remember also that to put a square root in surd form you take out the *biggest* square factor you can. Thus $\sqrt{48} = \sqrt{16} \times \sqrt{3} = 4\sqrt{3}$ (noting that you should take out $\sqrt{16}$ and not $\sqrt{4}$).

Youtube Videos to help!

Negative Powers – Exam Solutions - https://www.youtube.com/watch?v=SW9nb-13V6E

Fractional Powers – Exam Solutions - https://www.youtube.com/watch?v=_fadg_VjBMc

Rationalising Surds – Exam Solutions - https://www.youtube.com/watch?v=xehwCkT5aX0

Exercise 1.6

1 Write the following as powers of x.

(a) $\frac{1}{x}$ (b) $\frac{1}{x^5}$ (c) $\sqrt[5]{x}$ (d) $\sqrt[3]{x^5}$ (e) $\frac{1}{\sqrt{x}}$ (f) $\frac{1}{\sqrt{x^3}}$

2 Write the following without negative or fractional powers.

(a) x^{-4} (b) x^0 (c) $x^{1/6}$ (d) $x^{3/4}$ (e) $x^{-3/2}$

3 Write the following in the form ax^n.

(a) $4\sqrt[3]{x}$ (b) $\frac{3}{x^2}$ (c) $\frac{5}{\sqrt{x}}$ (d) $\frac{1}{2x^3}$ (e) 6

4 Write as sums of powers of x.

(a) $x^3\left(x+\frac{1}{x}\right)$ (b) $\frac{x^4+1}{x^2}$ (c) $x^{-5}\left(x+\frac{1}{x^2}\right)$

5 Write the following in surd form.

(a) $\sqrt{75}$ (b) $\sqrt{180}$ (c) $\frac{12}{\sqrt{6}}$ (d) $\frac{1}{\sqrt{5}}$ (e) $\frac{3}{\sqrt{12}}$

6 Rationalise the denominators in the following expressions.

(a) $\frac{1}{\sqrt{2}-1}$ (b) $\frac{2}{\sqrt{6}-2}$ (c) $\frac{6}{\sqrt{7}+2}$

(d) $\frac{1}{3+\sqrt{5}}$ (e) $\frac{1}{\sqrt{6}-\sqrt{5}}$

Further Maths Only

7* Simplify $\frac{1}{\sqrt{2}+\sqrt{1}}+\frac{1}{\sqrt{3}+\sqrt{2}}+\frac{1}{\sqrt{4}+\sqrt{3}}+\ldots+\frac{1}{\sqrt{100}+\sqrt{99}}$